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Tachyonic kink and lump-like solutions in superstring field theory

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Abstract: We construct a kink solution on a non-BPS D-brane using Berkovits' formulation of superstring field theory in the level truncation scheme. The tension of the kink reproduces 95% of the expected BPS D-brane tension. We also find a lump-like solution which is interpreted as a kink–antikink pair, and investigate some of its properties. These results may be considered as successful tests of Berkovits' superstring field theory combined with the modified level truncation scheme.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons, D-branes.

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Contents

1. Introduction

In a couple of years, Sen's conjectures on the tachyon condensation have successfully been examined using various types of open string field theories.[1] In Witten's cubic string field theory (CSFT) [2], not only have the (approximate) tachyon potential [3, 4] and lower dimensional D-branes as tachyonic lump solutions been con-

[1]For a review, see [1].

structed [5, 6], but also the absence of physical open string excitations (more precisely, the triviality of the cohomology classes) at the minimum of the tachyon potential has recently been verified in [7]. In boundary string field theory (BSFT) [8], the exact tachyon potential and lower dimensional D-branes have been worked out both in bosonic string theory [9] and in superstring theory ([10] for non-BPS D-branes and [11] for the brane-antibrane system). However, only a limited amount of works have been done using Berkovits' Wess-Zumino-Witten–like open superstring field theory [12]. So far, the universal tachyon potential [13]–[16], the effective potential for marginal deformation parameter of string field theory [17] and the tachyon potential in the D0/D4 system with a Neveu-Schwarz B-field [18] have been obtained. One of our aims of this paper is to provide this theory with a new piece of evidence that it correctly describes the dynamics of open superstrings by constructing tachyonic soliton solutions on a non-BPS Dp-brane which can be identified with BPS D$(p-1)$-branes. In more detail, we find a kink solution whose tension is 94.9% of that of a BPS D$(p-1)$-brane and a kink-antikink solution with its tension being 98.8% of the sum of the tensions of a BPS D$(p-1)$-brane and a BPS anti-D$(p-1)$-brane, in the level truncation scheme.

This paper is organized as follows. In section 2, after reviewing the formalism of Berkovits' superstring field theory, we prepare the level-expanded string field for the calculation of the action. In section 3, we describe the construction of the solitonic solutions and then compare their tensions with the expected results. In section 4 we briefly summarize our results. Some of the details about the calculations are collected in appendices A, B and C.

2. Superstring field theory

In this section we describe only the relevant parts of the structure of the theory for later calculations. For more details, see [14] and [1, chapter 3].

2.1 The action for the string field

The Wess-Zumino-Witten–like action for the Neveu-Schwarz sector string field $\widehat{\Phi}$ on a non-BPS Dp-brane of type II superstring theory is written as

$$\begin{aligned} S &= \frac{1}{4g_o^2}\left\langle\!\!\left\langle \left(e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}}\right)\left(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}\right) - \right.\right. \\ &\qquad\left.\left. - \int_0^1 dt\left(e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}}\right)\left\{\left(e^{-t\widehat{\Phi}}\widehat{Q}_B e^{t\widehat{\Phi}}\right),\left(e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}}\right)\right\}\right\rangle\!\!\right\rangle \qquad (2.1) \\ &= \frac{1}{2g_o^2}\sum_{M,N=0}^{\infty}\frac{(-1)^N}{(M+N+2)!}\binom{M+N}{N}\left\langle\!\!\left\langle\left(\widehat{Q}_B\widehat{\Phi}\right)\widehat{\Phi}^M\left(\widehat{\eta}_0\widehat{\Phi}\right)\widehat{\Phi}^N\right\rangle\!\!\right\rangle, \end{aligned}$$

holomorphic field	∂X^μ	ψ^μ	b	c	β	γ	$e^{\ell\phi}$	ξ	η	T	G
conformal weight h	1	1/2	2	-1	3/2	$-1/2$	$-\frac{1}{2}\ell^2-\ell$	0	1	2	3/2
ghost number $\#_{\rm gh}$	0	0	-1	$+1$	-1	$+1$	0	-1	$+1$	0	0
picture number $\#_{\rm pic}$	0	0	0	0	0	0	ℓ	$+1$	-1	0	0
world-sheet statistics	B	F	F	F	B	B	F(ℓ:odd) B(ℓ:even)	F	F	B	F

Table 1: Some properties of the fields on an $\mathcal{N}=1$ superstring world-sheet.

where

$$\widehat{Q}_B = (Q_0+Q_1+Q_2)\otimes\sigma_3\,, \qquad \widehat{\eta}_0 = \oint\frac{dz}{2\pi i}\eta(z)\otimes\sigma_3\,, \tag{2.2}$$

$$\begin{aligned} Q_0 &= \oint\frac{dz}{2\pi i}(cT^{\rm m} + c\,\partial\xi\,\eta + cT^\phi + c\,\partial c\,b)(z)\,,\\ Q_1 &= \oint\frac{dz}{2\pi i}\eta e^\phi G^{\rm m}(z)\,,\\ Q_2 &= -\oint\frac{dz}{2\pi i}\eta\,\partial\eta\,e^{2\phi}b(z)\,. \end{aligned} \tag{2.3}$$

The BRST charge Q_B was decomposed into three parts according to the ϕ-charge of each term. σ_3 is the *internal Chan-Paton* matrix explained below. And ξ,η,ϕ are the bosonized superconformal ghosts defined as

$$\beta = e^{-\phi}\partial\xi\,, \qquad \gamma = \eta e^\phi\,. \tag{2.4}$$

Notice the *orderings* of the factors above because ξ,η,e^ϕ are all fermionic. Some basic properties of world-sheet fields are summarized in table 1.

The operator product expansions (OPEs) among various fields on the disk boundary are

$$e^{ip\cdot X(z)}e^{iq\cdot X(w)} \sim |z-w|^{2\alpha' p\cdot q} : e^{ip\cdot X(z)}e^{iq\cdot X(w)} :, \tag{2.5}$$

$$\partial X^\mu(z)e^{ip\cdot X(w)} \sim -2i\alpha'\frac{p^\mu}{z-w}e^{ip\cdot X(w)}\,, \tag{2.6}$$

$$\partial X^\mu(z)\partial X^\nu(w) \sim -2\alpha'\frac{\eta^{\mu\nu}}{(z-w)^2}\,, \tag{2.7}$$

$$\psi^\mu(z)\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w}\,, \tag{2.8}$$

$$b(z)c(w) \sim \frac{1}{z-w}, \quad \xi(z)\eta(w) \sim \frac{1}{z-w}\,, \tag{2.9}$$

$$e^{\ell\phi(z)}e^{m\phi(w)} \sim (z-w)^{-\ell m} : e^{\ell\phi(z)}e^{m\phi(w)} :, \tag{2.10}$$

$$\partial\phi(z)e^{\ell\phi(w)} \sim -\frac{\ell}{z-w}e^{\ell\phi(w)}\,. \tag{2.11}$$

The energy momentum tensors and the supercurrent are

$$T^{\rm m}(z) = -\frac{1}{4\alpha'} : \partial X^\mu\partial X_\mu(z) : -\frac{1}{2} : \psi^\mu\partial\psi_\mu(z) :,$$

$$
\begin{aligned}
G^{\mathrm{m}}(z) &= \frac{i}{\sqrt{2\alpha'}}\psi^{\mu}\partial X_{\mu}(z)\,, \\
T^{\phi}(z) &= -\frac{1}{2} : \partial\phi\partial\phi(z) : -\partial^2\phi(z)
\end{aligned}
\tag{2.12}
$$

so that

$$
T^{\mathrm{m}}(z)T^{\mathrm{m}}(w) \sim \frac{15}{2(z-w)^4} + \frac{2}{(z-w)^2}T^{\mathrm{m}}(w) + \frac{1}{z-w}\partial T^{\mathrm{m}}(w)\,, \tag{2.13}
$$

$$
G^{\mathrm{m}}(z)G^{\mathrm{m}}(w) \sim \frac{10}{(z-w)^3} + \frac{2}{z-w}T^{\mathrm{m}}(w)\,. \tag{2.14}
$$

The string field is equipped with the following Chan-Paton structure

$$
\widehat{\Phi} = \Phi_+ \otimes I + \Phi_- \otimes \sigma_1 = \begin{pmatrix} \Phi_+ & \Phi_- \\ \Phi_- & \Phi_+ \end{pmatrix} \tag{2.15}
$$

according to the $e^{\pi i F}$ eigenvalue, where F measures the world-sheet fermion number. Due to this, the simple derivation properties

$$
\widehat{Q}_B\left(\widehat{\Phi}_1\widehat{\Phi}_2\right) = \left(\widehat{Q}_B\widehat{\Phi}_1\right)\widehat{\Phi}_2 + \widehat{\Phi}_1\left(\widehat{Q}_B\widehat{\Phi}_2\right), \quad \widehat{\eta}_0\left(\widehat{\Phi}_1\widehat{\Phi}_2\right) = \left(\widehat{\eta}_0\widehat{\Phi}_1\right)\widehat{\Phi}_2 + \widehat{\Phi}_1\left(\widehat{\eta}_0\widehat{\Phi}_2\right), \tag{2.16}
$$

of $\widehat{Q}_B$ and $\widehat{\eta}_0$ are preserved. In addition, they satisfy

$$
\left\{\widehat{Q}_B, \widehat{\eta}_0\right\} = 0\,, \qquad \widehat{Q}_B^2 = \widehat{\eta}_0^2 = 0\,. \tag{2.17}
$$

Finally, the 'bracket' $\langle\!\langle \ldots \rangle\!\rangle$ in eq. (2.1) is defined by

$$
\langle\!\langle \widehat{A}_1 \ldots \widehat{A}_n \rangle\!\rangle = \mathrm{Tr}_{\text{Chan-Paton}}\left\langle g_1^{(n)} \circ \widehat{A}_1(0) \ldots g_n^{(n)} \circ \widehat{A}_n(0)\right\rangle, \tag{2.18}
$$

where $\widehat{A}_i$'s are arbitrary vertex operators with Chan-Paton matrices. $g_k^{(n)}(z)$ is geometrically the conformal transformation which maps an upper half disk representing the k-th open string world-sheet to the k-th $(360/n)^\circ$ wedge belonging to the global unit disk realizing the Witten's associative gluing interaction among open strings, and is concretely defined to be

$$
g_k^{(n)}(z) = e^{\frac{2\pi i}{n}(k-1)}\left(\frac{1+iz}{1-iz}\right)^{2/n} \qquad \text{for } \ 1 \le k \le n. \tag{2.19}
$$

For later convenience, we record

$$
g_k^{(n)\prime}(0) = \frac{4i}{n}e^{2\pi i\frac{k-1}{n}}\,, \tag{2.20}
$$

$$
g_k^{(n)\prime\prime}(0) = \left(\frac{4i}{n}\right)^2 e^{2\pi i\frac{k-1}{n}}\,. \tag{2.21}
$$

$g_k^{(n)} \circ A$ appearing in eq. (2.18) denotes the conformal transform of A by $g_k^{(n)}$ and is *naïvely* written as

$$g_k^{(n)} \circ A(0) \simeq \Big(g_k^{(n)\prime}(0)\Big)^h A\Big(g(0)\Big)\,, \tag{2.22}$$

where we assumed that A is a primary field of conformal weight h. A problem arises when h is non-integral. In fact, we will later deal with vertex operators having non-zero momenta, typically written as $A = A_0 e^{ipX}$. Dividing the weight h into two parts as

$$h = \alpha' p^2 + h_N\,,$$

where h_N denotes the contribution from A_0, we unambiguously define the conformal transform as

$$g_k^{(n)} \circ (A_0 e^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2 + h_N}\right| e^{2\pi i h_N\left(\frac{k-1}{n}+\frac{1}{4}\right)} A_0 e^{ipX}\Big(g_k^{(n)}(0)\Big)\,, \tag{2.23}$$

where use was made of eq. (2.20). $\langle \ldots \rangle$ in eq. (2.18) then represents the correlation function in the combined matter-ghost superconformal field theory on the unit disk with n conformally transformed vertex operators inserted on the boundary of the disk. It is normalized as

$$\left\langle \xi(z_1)\frac{1}{2}c\partial c\partial^2 c(z_2) e^{-2\phi(z_3)}\right\rangle = 1\,, \tag{2.24}$$

which is independent of (z_1, z_2, z_3) because the above insertions play the rôle of filling the zero-modes. Note that the string field theory under consideration is formulated in the "large" Hilbert space including the ξ zero mode, and that this fact is taken into account in the normalization (2.24).

Here we write down some important properties which $\langle\langle \ldots \rangle\rangle$ possesses:[2]

$$\begin{aligned} \langle\langle A_1 \ldots A_{n-1}\Phi\rangle\rangle &= \langle\langle \Phi A_1 \ldots A_{n-1}\rangle\rangle\,, \\ \langle\langle A_1 \ldots A_{n-1}(Q_B\Phi)\rangle\rangle &= -\langle\langle (Q_B\Phi) A_1 \ldots A_{n-1}\rangle\rangle, \qquad \text{(cyclicity)} \qquad (2.25) \\ \langle\langle A_1 \ldots A_{n-1}(\eta_0\Phi)\rangle\rangle &= -\langle\langle (\eta_0\Phi) A_1 \ldots A_{n-1}\rangle\rangle\,, \\ \langle\langle Q_B(\ldots)\rangle\rangle &= \langle\langle \eta_0(\ldots)\rangle\rangle = 0\,. \qquad (2.26) \end{aligned}$$

In the last line of eq. (2.1), we have already exploited the cyclicity properties implicitly. Thanks to the relations (2.26), we can carry out 'partial integration' which technically facilitates the actual computations of correlators. Now, we are ready to compute the string field theory action (2.1) if we are given an explicit representation of the string field $\widehat{\Phi}$ in terms of vertex operators.

[2] For the proofs of these relations see [14, 1].

2.2 Level expansion of the string field on a non-BPS D-brane

In this paper, we will search for codimension 1 soliton solutions to the equations of motion derived from the action (2.1) in the level truncation scheme. Let us consider the open superstring field theory on a non-BPS Dp-brane, and pick out one particular direction, say $X^9 \equiv X$, tangential to the original Dp-brane along which we will develop non-trivial field configurations. We denote by $\mathcal{M}$ the remaining 9-dimensional manifold. Since we require the string field configuration to have zero momentum in any direction along $\mathcal{M}$, we can consistently restrict the string field to the universal subspace of the full string Hilbert space when focusing on $\mathcal{M}$: This subspace is spanned by linear combinations of states obtained by acting on the zero-momentum oscillator vacuum with the ghost oscillators $b_n, c_n, \beta_r, \gamma_r$ and the matter super-Virasoro generators $L_n^{\mathcal{M}}, G_r^{\mathcal{M}}$. This subspace is universal because it has no dependence on the details of the boundary conformal field theory describing the D-brane [19]. Along the above discussions, we decompose the full matter super-Virasoro generators as

$$L_n^{\mathrm{m}} = L_n^X + L_n^{\mathcal{M}}\,, \qquad G_r^{\mathrm{m}} = G_r^X + G_r^{\mathcal{M}}\,, \tag{2.27}$$

the meaning of which would be obvious. But we must pay attention to the following OPE

$$G^{\mathcal{M}}(z)G^{\mathcal{M}}(w) \sim \frac{9}{(z-w)^3} + \frac{2}{z-w}T^{\mathcal{M}}(w)\,. \tag{2.28}$$

Regarding the X-direction, however, the string field is forced to lie outside the universal class, so we choose the oscillator basis instead of the universal basis in the matter sector too.

Here we define the *level* of a state $|\Phi, p\rangle$ with momentum p to be

$$\text{level} = h + \frac{1}{2} = \alpha' p^2 + h_N + \frac{1}{2}\,, \tag{2.29}$$

where the conformal weight h is the L_0^{tot} eigenvalue. Namely, we adopt the 'modified' definition of the level introduced in [6]. And the level of a component field is defined to be equal to that of the associated state. According to [6], we compactify the X-direction on a circle of radius R, so that $X \sim X + 2\pi R$ and that the momentum is quantized as $p = n/R$ with n taking (half-)integer values. To this end we calculate the action using the string field truncated up to level $3/2 + \alpha'/R^2$. Making use of the twist symmetry, it turns out that component fields of oscillator level 1/2 and 1 need not acquire non-vanishing expectation values in constructing soliton solutions [14], where by 'oscillator level' we mean the value of the level excluding the contribution from momentum, i.e. $h_N + 1/2$. So we set these fields to zero altogether from the beginning. We further diminish the number of states by imposing the Feynman-Siegel–ξ gauge conditions on the string field,

$$b_0|\Phi\rangle = \xi_0|\Phi\rangle = 0\,. \tag{2.30}$$

Though these gauge conditions are valid at the linearized level as shown in [4] and in [1, section 2.4 and 3.6], it is not clear whether they are also good even non-perturbatively. As to the lump solution[3] in bosonic cubic string field theory found in [6], the validity of the Feynman-Siegel gauge condition was verified in [21] by checking that the lump solution obtained *in* the Feynman-Siegel gauge actually solves the full set of equations of motion derived from the gauge invariant (i.e. not gauge-fixed) string field theory action. Here we use the gauge conditions (2.30) by simply assuming the validity of this gauge in constructing the classical solutions. Legitimacy of b_0 gauge condition $b_0|\Phi\rangle = 0$ can probably be verified in the same way as in [21] by including the states which do not satisfy $b_0|\Phi\rangle = 0$ (i.e. the states containing c_0 oscillator) and examining whether our solutions solve the full set of equations of motion derived from the gauge invariant action. Getting rid of the ξ_0 gauge condition, however, is more difficult because it breaks the one-to-one correspondence between the states in the "small" Hilbert space and those in the "large" Hilbert space. Answering this question in this superstring field theory remains open.

Putting the above accounts together, we can write down the level-truncated string field in the following way. First, required states in the "small" Hilbert space are

$$\mathcal{T}_n = |\tilde{\Omega}, n\rangle = c_1 e^{-\phi(0)} e^{i\frac{n}{R}X(0)}|0\rangle \qquad \text{at oscillator level } 0\,, \tag{2.31}$$

$$\begin{aligned} \mathcal{A}_n &= c_{-1}\beta_{-1/2}|\tilde{\Omega}, n\rangle\,, & \mathcal{E}_n &= b_{-1}\gamma_{-1/2}|\tilde{\Omega}, n\rangle\,, \\ \mathcal{F}_n &= G^{\mathcal{M}}_{-3/2}|\tilde{\Omega}, n\rangle\,, & \mathcal{G}_n &= \beta_{-1/2}\gamma_{-1/2}\psi^X_{-1/2}|\tilde{\Omega}, n\rangle\,, \\ \mathcal{H}_n &= \psi^X_{-3/2}|\tilde{\Omega}, n\rangle\,, & \mathcal{J}_n &= \psi^X_{-1/2}\alpha^X_{-1}|\tilde{\Omega}, n\rangle\,, \end{aligned} \tag{2.32}$$

where $|0\rangle$ is the SL_2-invariant vacuum, and all these states have ghost number 1 and picture number -1. The last six states are at oscillator level 3/2. As mentioned earlier, we adopt the complete oscillator basis in the X-direction, whereas in the $\mathcal{M}$-direction the universal Virasoro basis is employed. When they are translated into the vertex operator representation in the "large" Hilbert space, the ξ_0 gauge condition (2.30) allows us to map any state in the "small" Hilbert space to a certain vertex operator in the "large" one in a one-to-one way. That is, the rule of translation is simply given by

$$\underset{\text{state}}{\mathcal{T}_n = c_1 e^{-\phi(0)} e^{i\frac{n}{R}X(0)}|0\rangle} \longrightarrow \underset{\text{"small"}}{c e^{-\phi} e^{i\frac{n}{R}X}} \longrightarrow \underset{\text{"large"}}{T_n = \xi c e^{-\phi} e^{i\frac{n}{R}X}} \tag{2.33}$$

for the tachyon state, for example. Using this rule, the string field $\widehat{\Phi}$ takes the following form,

$$\begin{aligned} \widehat{\Phi} &= T \otimes \sigma_1 + (E + V + A) \otimes I\,; \\ V &= G + F + H + J\,, \end{aligned} \tag{2.34}$$

[3] For the spatially homogeneous 'closed string vacuum' solution of [4, 3], the validity of the Feynman-Siegel gauge condition was established in [20].

where

$$
\begin{aligned}
T &= \sum_n t_n T_n = \sum_n t_n \ \xi c e^{-\phi} e^{i\frac{n}{R}X} \,, \\
E &= \sum_n e_n E_n = \sum_n e_n \ \xi \eta e^{i\frac{n}{R}X} \,, \\
A &= \sum_n a_n A_n = \sum_n a_n \ \xi \partial \xi c \partial^2 c e^{-2\phi} e^{i\frac{n}{R}X} \,, \\
G &= \sum_n g_n G_n = \sum_n g_n \ \xi c (\partial e^{-\phi}) \psi^X e^{i\frac{n}{R}X} \,, \\
F &= \sum_n f_n F_n = \sum_n f_n \ \xi c e^{-\phi} G^{\mathcal{M}} e^{i\frac{n}{R}X} \,, \\
H &= \sum_n h_n H_n = \sum_n h_n \ \xi c e^{-\phi} (\partial \psi^X) e^{i\frac{n}{R}X} \,, \\
J &= \sum_n j_n J_n = \frac{i}{\sqrt{\alpha'}} \sum_n j_n \ \xi c e^{-\phi} \psi^X \partial X e^{i\frac{n}{R}X} \,.
\end{aligned}
\tag{2.35}
$$

Since ξ has ghost number -1 and picture number 1, the string field $\widehat{\Phi}$ is expressed by vertex operators of ghost number 0 and picture number 0, as required. The tachyon vertex operator T is tensored by the intenal Chan-Paton matrix σ_1 because the Neveu-Schwarz sector ground state $|\tilde{\Omega}, n\rangle$ itself is in the GSO($-$) sector, while the other states appearing in (2.32) lie in the GSO(+) sector so that they are multiplied by I. Since E, A, G, H and J are not conformal primary fields, their conformal properties are complicated: we record them, as well as the action of the BRST charge on the vertex operators, in appendix A.

2.3 Calculation of the action

Now that we have explained all ingredients, we can in principle calculate the string field theory action (2.1) and express it in terms of the component fields $\{t_n, e_n, a_n, g_n, f_n, h_n, j_n\}$ by substituting the string field (2.34) into the action (2.1), carrying out the conformal transformations, acting on the vertex operators with Q_B and η_0, and evaluating the correlators on the disk using the OPEs (2.5)–(2.11), (2.28) and the normalization (2.24). But unfortunately, computations are tedious and even the final expressions of the action are awfully lengthy. So we present only some representative terms in appendix B.

3. Solutions and their properties

In this section, we search for solutions which non-trivially depend on the coordinate in the X-direction by extremizing the action written in terms of a finite number of component fields. For a technical reason, we deal with the case of a single kink and the case of a kink-antikink pair separately in subsection 3.1 and 3.2, respectively.

3.1 Kink solution

First of all, we must define 'what is the kink solution'. Since the matter part of the tachyon vertex operator has the form

$$T^{\text{matter}} = \sum_n t_n e^{i\frac{n}{R}X} ,$$

we can naturally consider the following function

$$t(x) = \sum_n t_n e^{i\frac{n}{R}x} \tag{3.1}$$

of the center-of-mass coordinate x as representing the profile of the tachyon field on the original non-BPS Dp-brane, as is done in [6]. The reality condition $t(x)^* = t(x)$ of the tachyon field demands that its Fourier components $\{t_n\}$ should satisfy

$$t_n^* = t_{-n} . \tag{3.2}$$

Since the tachyon potential on a non-BPS D-brane has doubly-degenerate minima $\pm\bar{t}$, we could obviously define a tachyonic kink configuration, if the X-direction was non-compact $\mathbf{R}$, to be

$$t(x) \longrightarrow \begin{cases} \bar{t} & \text{for } x \to +\infty \\ -\bar{t} & \text{for } x \to -\infty \end{cases} , \tag{3.3}$$

and similarly an antikink configuration with the signs reversed. But now, we are compactifying the X-direction on a circle of radius R, so that $x \in \mathcal{I} = [-\pi R, \pi R]$. In this case, we refer to a configuration which obeys

$$t(x) \sim \begin{cases} \bar{t} & \text{near } \; x = \pi R \\ -\bar{t} & \text{near } x = -\pi R \end{cases} \tag{3.4}$$

and is a monotonically increasing function of $x \in \mathcal{I}$ as a tachyonic *kink*. From eq. (3.4), we are immediately aware that the tachyon field $t(x)$ must satisfy *antiperiodic* boundary condition. This is achieved by letting the discrete momentum n take value in $\mathbb{Z} + \frac{1}{2}$. Physically, it corresponds to turning on a $\mathbb{Z}_2$ Wilson line along the circle so that the boundary condition is twisted [22, 6]. For ease of calculations, we will look for a kink solution whose profile is an odd function of $x \in \mathcal{I}$. This condition gives

$$t(x) = -t(-x) \quad \longrightarrow \quad t_{-n} = -t_n . \tag{3.5}$$

This constraint, combined with (3.2), means that every t_n must be purely imaginary, $t_n^* = -t_n$. Hence we set

$$t_n = -t_{-n} = \frac{1}{2i}\tau_n \tag{3.6}$$

with τ_n real so that

$$t(x) = \sum_{n\in\mathbb{Z}+\frac{1}{2}} t_n e^{i\frac{n}{R}x} = \sum_{n\in\mathbb{Z}_{\geq 0}+\frac{1}{2}} \tau_n \sin\frac{n}{R}x . \tag{3.7}$$

3.1.1 Kink solution in purely tachyonic superstring field theory

By 'purely tachyonic superstring field theory', we mean the one in which the string field $\widehat{\Phi}$ contains only the tachyon state and its harmonics, namely the superstring field theory truncated to the oscillator level 0. In this theory, the action involves only those terms which are quadratic and quartic in the tachyon field, so that the action can be calculated fairly easily. The result is

$$\begin{aligned} 2g_o^2 S_T &= \frac{1}{2}\left\langle\!\!\left\langle \left(\widehat{Q}_B\widehat{T}\right)\left(\widehat{\eta}_0\widehat{T}\right)\right\rangle\!\!\right\rangle + \frac{1}{12}\left(\left\langle\!\!\left\langle \left(\widehat{Q}_B\widehat{T}\right)\widehat{T}^2\left(\widehat{\eta}_0\widehat{T}\right)\right\rangle\!\!\right\rangle - \left\langle\!\!\left\langle \left(\widehat{Q}_B\widehat{T}\right)\widehat{T}\left(\widehat{\eta}_0\widehat{T}\right)\widehat{T}\right\rangle\!\!\right\rangle\right) \\ &= 2\pi R V_p\left\{\sum\left(-\frac{\alpha'}{R^2}n^2+\frac{1}{2}\right)t_n t_{-n} - \right. \qquad (3.8) \\ &\quad \left. -\sum \delta_{n_1+n_2+n_3+n_4} t_{n_1}t_{n_2}t_{n_3}t_{n_4}\exp\left[\frac{\alpha'}{R^2}\log 2\left((n_1+n_3)^2-\sum_{I=1}^{4}(n_I)^2\right)\right]\right\}, \end{aligned}$$

where V_p is the volume of p-dimensional subspace of the original Dp-brane perpendicular to the X-direction. The details of the calculations are shown in appendix B. In order to find a numerical solution, we must assign a numerical value to the radius R. In this subsection, we choose

$$R = \sqrt{\frac{77}{6}\alpha'} \qquad (3.9)$$

for a reason mentioned in the next sub-subsection. Given this value, the level of each Fourier mode t_n of the tachyon field is found to be

$$\mathrm{level}(t_n) = \mathrm{level}(\tau_n) = \frac{\alpha'}{R^2}n^2 = \frac{6}{77}n^2\,, \qquad (3.10)$$

and we keep only those modes with level equal to or less than $\frac{243}{154} = \frac{3}{2} + \frac{6}{77}$. In other words, the summation is taken over $|n| \le 9/2$. Furthermore, we truncate the action at level 243/77, which means that in the quartic interaction terms in eq.(3.8) we discard those terms with level

$$\frac{6}{77}(n_1^2+n_2^2+n_3^2+n_4^2)$$

exceeding 243/77. That is, we adopt the level $(\frac{3}{2}+\frac{\alpha'}{R^2}, 3+2\frac{\alpha'}{R^2}) = (\frac{243}{154}, \frac{243}{77})$ approximation to the action. This procedure gives the following action,[4]

$$\begin{aligned} 2g_o^2 S_T^{\left(\frac{243}{154},\frac{243}{77}\right)} = -\frac{2\pi R V_p}{\pi^2}\Big(&-2.37\tau_{1/2}^2 + 3.57\tau_{1/2}^4 - 4.43\tau_{1/2}^3\tau_{3/2} - 1.60\tau_{3/2}^2 + \\ &+12.42\tau_{1/2}^2\tau_{3/2}^2 + 2.75\tau_{3/2}^4 - 10.74\tau_{1/2}^2\tau_{3/2}\tau_{5/2} + 10.18\tau_{1/2}\tau_{3/2}^2\tau_{5/2} - \\ &-0.0641\tau_{5/2}^2 + 9.46\tau_{1/2}^2\tau_{5/2}^2 + 8.73\tau_{3/2}^2\tau_{5/2}^2 + 1.87\tau_{5/2}^4 - \end{aligned}$$

[4] In the original version of this paper, we had dropped the factor of $-\frac{1}{\pi^2}$ in eq.(3.11).

$$
\begin{aligned}
&-8.09\tau_{1/2}\tau_{3/2}^2\tau_{7/2} - 7.62\tau_{1/2}^2\tau_{5/2}\tau_{7/2} + 14.83\tau_{1/2}\tau_{3/2}\tau_{5/2}\tau_{7/2} + \\
&+6.85\tau_{3/2}\tau_{5/2}^2\tau_{7/2} + 2.24\tau_{7/2}^2 + 6.39\tau_{1/2}^2\tau_{7/2}^2 + 6.29\tau_{3/2}^2\tau_{7/2}^2 + \\
&+6.05\tau_{5/2}^2\tau_{7/2}^2 - 1.87\tau_{3/2}^3\tau_{9/2} - 10.80\tau_{1/2}\tau_{3/2}\tau_{5/2}\tau_{9/2} + \\
&+5.26\tau_{1/2}\tau_{5/2}^2\tau_{9/2} - 4.82\tau_{1/2}^2\tau_{7/2}\tau_{9/2} + 9.75\tau_{1/2}\tau_{3/2}\tau_{7/2}\tau_{9/2} + 5.32\tau_{9/2}^2\Big) \,,
\end{aligned}
\tag{3.11}
$$

where we used eq. (3.6). Since the action has been rewritten as a simple quartic polynomial of five variables $(\tau_{1/2}, \tau_{3/2}, \tau_{5/2}, \tau_{7/2}, \tau_{9/2})$, we can easily extremize it by solving simultaneous equations of motion $\{\partial S_T/\partial\tau_n = 0\}$ numerically. With a suitable choice of initial values for τ_n's, our numerical algorithm has converged to

$$
\begin{aligned}
\Phi_k = \Big\{&\tau_{1/2} = 0.626558, \tau_{3/2} = 0.180632, \tau_{5/2} = 0.0745727\,, \\
&\tau_{7/2} = 0.0264652, \tau_{9/2} = 0.00983679\Big\}\,.
\end{aligned}
\tag{3.12}
$$

Substituting these values into eq. (3.7) we can find the tachyon profile, which is plotted in figure 1.

This figure nicely shows that the solution (3.12) corresponds to the desired tachyonic kink configuration. In particular, the tachyon field correctly approaches the vacuum values in the asymptotic region $x \sim \pm\pi R$.

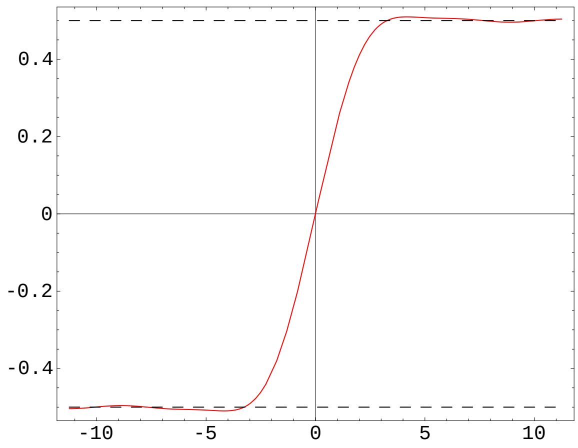

Figure 1: The solid line shows a plot of $t(x)$ found at the purely tachyonic level. The dashed lines indicate the vacuum expectation values $\pm\bar{t} = \pm 0.5$ of the tachyon field at the minima of the level $(0,0)$ tachyon potential.

The next step is to calculate the tension of the kink solution. First, we reexpress the open string coupling constant g_o defined for open superstrings on the original non-BPS Dp-brane in terms of the tension $\tilde{\tau}_p$ of the non-BPS Dp-brane. This relation has already been worked out in [14] as

$$
\tilde{\tau}_p = \frac{1}{2\pi^2 g_o^2}\,. \tag{3.13}
$$

Using it, the purely tachyonic action (3.8) is rewritten as

$$
S_T = 2\pi R V_p \tilde{\tau}_p \pi^2 \left\{ \sum \left(-\frac{\alpha'}{R^2} n^2 + \frac{1}{2} \right) t_n t_{-n} + \cdots \right\}. \tag{3.14}
$$

Since the action on a non-BPS Dp-brane is proportional to the $(p+1)$-dimensional volume $2\pi R V_p$ and the brane tension $\tilde{\tau}_p$, we generally define

$$
f(\Phi) = -\frac{1}{(2\pi R V_p)\tilde{\tau}_p} S(\Phi) = -\frac{\pi^2}{2\pi R V_p} 2 g_o^2 S(\Phi)\,. \tag{3.15}
$$

As the additive normalization of the string field theory action (2.1) is given by $S(0) = 0$, the Sen's conjecture about the brane annihilation is stated as $f(\Phi_0) = -1$, where Φ_0 denotes the 'closed string vacuum' configuration. In order to calculate the kink tension, however, we have to add a suitable constant term to the tachyon potential such that the energy density at the bottom of the potential should vanish. At the purely tachyonic level, this can be done by shifting $f(\Phi)$ by $f^{(0,0)}(\Phi_0) \simeq -0.617$, where $f^{(0,0)}$ is the level $(0,0)$ tachyon potential [13]. Then the value of the shifted action evaluated on the kink solution (3.12) becomes

$$\mathcal{S}_T^{\left(\frac{243}{154},\frac{243}{77}\right)} = 2\pi R V_p \tilde{\tau}_p \left(f^{(0,0)}(\Phi_0) - f^{\left(\frac{243}{154},\frac{243}{77}\right)}(\Phi_k) \right) \equiv -V_p \mathcal{T}_{p-1} \,, \tag{3.16}$$

where we have denoted the kink tension by $\mathcal{T}_{p-1}$. Since it is conjectured (and in fact verified in the context of boundary superstring field theory [10]) that a tachyonic kink configuration on a non-BPS Dp-brane is identified with a BPS D$(p-1)$-brane with tension $\tau_{p-1} = 2\pi\sqrt{\alpha'}\tilde{\tau}_p/\sqrt{2}$, we need to calculate the ratio

$$r = \frac{\mathcal{T}_{p-1}}{\tau_{p-1}} = \frac{\sqrt{2}\mathcal{T}_{p-1}}{2\pi\sqrt{\alpha'}\tilde{\tau}_p} = \sqrt{2}\frac{R}{\sqrt{\alpha'}} \left(f^{\left(\frac{243}{154},\frac{243}{77}\right)}(\Phi_k) - f^{(0,0)}(\Phi_0) \right) \tag{3.17}$$

and to see whether the value of r is in the vicinity of unity. Putting the numerical values (3.12) into the above expression and evaluating it, we obtain

$$r \simeq 0.640 \,. \tag{3.18}$$

Thus we conclude that the tension of the kink solution in the purely tachyonic superstring field theory is about 64% of the expected answer, for a particular choice (3.9) of radius. Such a degree of accuracy, which is slightly better than that of the depth of the tachyon potential, is quite reasonable when we compare it with the result of the lump solutions in the bosonic cubic string field theory [6]. Although it is interesting to see whether or not the value of r is approximately independent of the chosen values of the radius R, we leave it for future studies.

3.1.2 Including level 3/2 fields

We have seen in the last sub-subsection that using the purely tachyonic theory we qualitatively succeed in constructing a tachyonic kink solution. Nevertheless, the value $r \simeq 0.640$ is not so satisfactory. In this sub-subsection we will be looking for a kink solution with a higher accuracy by including oscillator level 3/2 fields.

We begin by determining the boundary conditions, or modings, of component fields. By definition of the tachyonic kink, the tachyon field $t(x)$ must obey the antiperiodic boundary condition. Therefore, the tachyon field is again expanded in half-integral modes. Other fields at oscillator level 3/2, however, turn out to satisfy the *periodic* boundary condition.[5] To see this, let us regard the non-BPS Dp-brane

[5] I thank T. Takayanagi for pointing this out.

and various fields on it as being obtained by modding out the coincident Dp-brane-anti-Dp-brane system by the action of $(-1)^{F_L}$ [23]. Then the tachyonic mode arises in the spectrum of anti-GSO-projected p-$\bar{p}$ strings. Turning on a $\mathbb{Z}_2$ Wilson line along the X-direction of one of the two branes, the boundary condition of the tachyon field, which is in the bifundamental representation of the gauge group $\mathrm{U}(1)\times\mathrm{U}(1)$, is reversed. On the other hand, level 3/2 fields living in the GSO(+) sector all come from p-p or $\bar{p}$-$\bar{p}$ strings. Since such fields are neutral under the gauge group, their boundary conditions are not affected by the existence of the $\mathbb{Z}_2$ Wilson line. As a result, these fields have integer modes.

Now we proceed to level expansion of the string field. For $R=\sqrt{77\alpha'/6}$ the first harmonics ($|n|=1$ modes) of the oscillator level 3/2 fields are at level

$$\frac{3}{2}+\frac{\alpha'}{R^2}=\frac{243}{154}.$$

As we have already seen before, this value coincides with the level number of the $|n|=9/2$ modes of the tachyon field: This is why we have chosen the strangely-looking value $R=\sqrt{77\alpha'/6}$ as the radius. Truncating the string field and the action at level $\left(\frac{243}{154},\frac{243}{77}\right)$, one obtains the approximate action as a sixth-order polynomial of 23 variables, though its full expression is quite long. We have searched for a stationary point of this action by solving numerically 23 simultaneous equations of motion obtained by differentiating the action with respect to the 23 variables. Starting from the results (3.12) obtained in the purely tachyonic theory, we have reached the following set of values using our algorithm,

$$\Phi_k=\left\{\begin{array}{lll} \tau_{1/2}=0.733584, & \tau_{3/2}=0.201643, & \tau_{5/2}=0.0915225, \\ \tau_{7/2}=0.0363576, & \tau_{9/2}=0.0111283, & \\ e_0=0.0829011, & e_1=-0.00933999, & e_{-1}=-0.00933999, \\ a_0=0.0445132, & a_1=-0.00978734, & a_{-1}=-0.00978734, \\ f_0=0.00996780, & f_1=-0.00219249, & f_{-1}=-0.00219249, \\ g_0=-2.53\times10^{-19}, & g_1=-0.000979490, & g_{-1}=0.000979490, \\ h_0=2.15\times10^{-19}, & h_1=0.00251080, & h_{-1}=-0.00251080, \\ j_0=-0.00416412, & j_1=-0.0103785, & j_{-1}=-0.0103785 \end{array}\right\}. \tag{3.19}$$

The tachyon profile (3.7) is plotted in figure 2. Though we can hardly distinguish it from figure 1, one should note that the dashed lines in figure 2 signify the vacuum values of the tachyon field at the minima of the level $\left(\frac{3}{2},3\right)$ tachyon potential. The fact that the tachyon field representing the kink solution correctly asymptotes to the vacuum values suggests that our action, as well as the superstring field theory, is right. Moreover, the tension of this kink solution can be calculated by substituting the values (3.19) into the formula

$$r=\sqrt{2}\frac{R}{\sqrt{\alpha'}}\left(f^{\left(\frac{243}{154},\frac{243}{77}\right)}(\Phi_k)-f^{\left(\frac{3}{2},3\right)}(\Phi_0)\right), \tag{3.20}$$

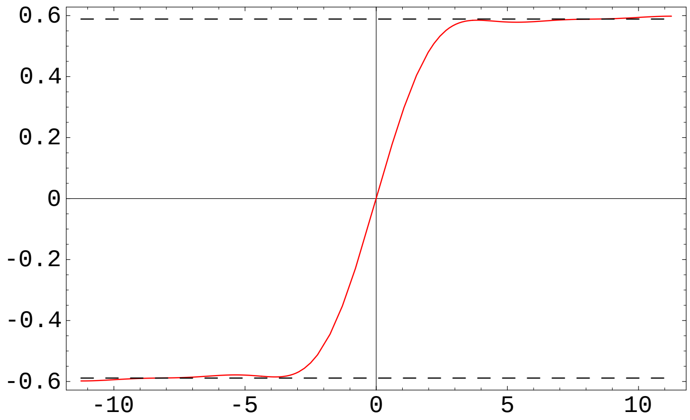

Figure 2: The solid line shows a plot of $t(x)$ found in the level $\left(\frac{243}{154}, \frac{243}{77}\right)$ truncated superstring field theory. The dashed lines indicate the vacuum expectation values $\pm\bar{t} = \pm 0.58882$ of the tachyon field at the minima of the level $\left(\frac{3}{2}, 3\right)$ tachyon potential.

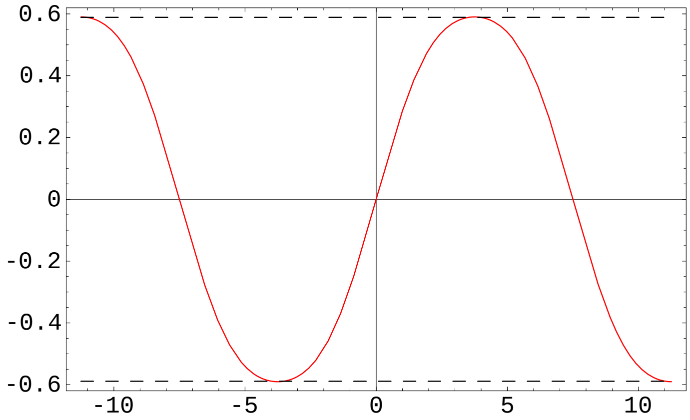

Figure 3: The solid line shows a plot of $t(x)$ in the 'triple brane' solution.

where $f(\Phi)$ is defined in eq. (3.15) and the depth of the tachyon potential at level $\left(\frac{3}{2}, 3\right)$ is given by [14]

$$f^{\left(\frac{3}{2},3\right)}(\Phi_0) \simeq -0.854\,.$$

The resulting value of the ratio r is found to be

$$r \simeq 0.949\,.$$

We can say that this value (about 95% of the expected one) is considerably better than the previous one.

3.1.3 An extra solution: triple brane

In the course of the search for the kink solution, we have encountered a brane-antibrane-brane configuration (figure 3) whose tension is about 92% of three times the BPS D$(p-1)$-brane tension.

3.2 Kink-antikink pair

In this subsection we will repeat essentially the same analysis as in the last subsection, with $t(x)$ replaced by an even function of $x \in \mathcal{I}$ satisfying the periodic boundary condition. $t(x) = t(-x)$ combined with the reality condition (3.2) implies

$$t_n = t_{-n} = t_n^* \equiv \tau_n \,. \tag{3.21}$$

Hence the tachyon profile is now given by

$$t(x) = \sum_{n\in\mathbb{Z}} t_n e^{i\frac{n}{R}x} = \tau_0 + 2\sum_{n=1}^{(\infty)} \tau_n \cos\frac{n}{R}x \,. \tag{3.22}$$

3.2.1 Lump-like solution in purely tachyonic superstring field theory

In this subsection our choice of radius[6] is

$$R = 4\sqrt{\alpha'} \tag{3.23}$$

for the same reason as mentioned in the previous subsection. Then the level of the tachyon field becomes

$$\text{level}(t_n) = \text{level}(\tau_n) = \frac{\alpha'}{R^2}n^2 = \frac{n^2}{16} \,. \tag{3.24}$$

Truncating the field and the action at level $\left(\frac{25}{16}, \frac{25}{8}\right)$ and solving the equations of motion, we have obtained the following solution

$$\Phi_\ell = \left\{\tau_0 = 0., \tau_1 = 0.301978, \tau_2 = 0., \tau_3 = -0.0560825, \tau_4 = 0., \tau_5 = 0.00860546\right\} \tag{3.25}$$

with the profile shown in figure 4. Since this lump-like solution can be interpreted as representing a kink-antikink pair, we should compare the tension of the solution, calculated as in eq. (3.16), with the sum of the tensions of a BPS D$(p-1)$-brane and a BPS anti-D$(p-1)$-brane.[7] Hence the ratio r (3.17) in this case should be modified as

$$r = \frac{\mathcal{T}_{p-1}}{2\times\tau_{p-1}} = \frac{1}{\sqrt{2}}\frac{R}{\sqrt{\alpha'}}\left(f^{\left(\frac{25}{16},\frac{25}{8}\right)}(\Phi_\ell) - f^{(0,0)}(\Phi_0)\right) . \tag{3.26}$$

Substituting the numerical values (3.25) into the above formula, we get

$$r \simeq 0.639 \,,$$

which happens to be very close to the previous result (3.18) for a kink.

[6]In [6] the authors avoided using rational values for $R/\sqrt{\alpha'}$ in order for the Verma module to agree with the total space spanned by the complete oscillator basis in the nonzero momentum sectors. This consideration does not matter to us here because we are adopting the oscillator basis (2.32) in the X-direction from beginning to end.

[7]One may think that the attractive force between a brane and an antibrane lowers the energy of the total system: This issue will be discussed in sub-subsection 3.2.3.

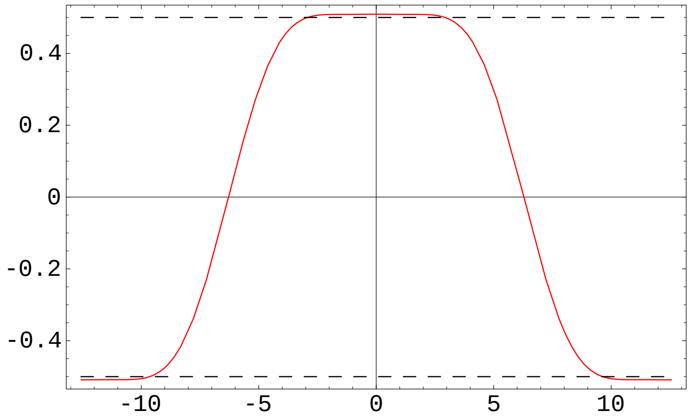

Figure 4: The solid line shows a plot of $t(x)$ found at the purely tachyonic level. The dashed lines indicate the vacuum expectation values $\pm\bar{t} = \pm 0.5$ of the tachyon field at the minima of the level $(0,0)$ tachyon potential.

3.2.2 Including level 3/2 fields

For $R = 4\sqrt{\alpha'}$, the level of the first harmonics ($|n| = 1$) of the oscillator level 3/2 fields is

$$\frac{3}{2} + \frac{\alpha'}{R^2} = \frac{25}{16},$$

which coincides with that of the $|n| = 5$ modes of the tachyon field. By adopting the level $\left(\frac{25}{16}, \frac{25}{8}\right)$ approximation to the action, it can be written as a polynomial of 24 variables. Solving the equations of motion gives

$$\Phi_\ell = \left\{ \begin{array}{lll} \tau_0 = -9.06 \times 10^{-12}, & \tau_1 = 0.344976, & \tau_2 = 6.77 \times 10^{-12}, \\ \tau_3 = -0.0703883, & \tau_4 = -2.79 \times 10^{-12}, & \tau_5 = 0.00985764, \\ e_0 = 0.0752118, & a_0 = 0.0348621, & f_0 = 0.00760330, \\ j_0 = -0.0146609, & |\text{other 14 modes}| < 10^{-12} & \end{array} \right\}. \tag{3.27}$$

Note that the above solution says τ_0, τ_2, τ_4 are practically *zero*. It then follows that the tachyon profile (3.22) crosses the horizontal axis at $x = \pm\pi R/2$, as shown in figure 5. The physical meaning of this fact is, interestingly, that a D$(p-1)$-brane and an anti-D$(p-1)$-brane, constructed as a kink and an antikink on a non-BPS Dp-brane respectively, are located at diametrically opposite points of the circle in the compactified X-direction. What we have to mention is that our numerical algorithm automatically converges to the solution (3.27) even if we give non-zero initial values to τ_0, τ_2, τ_4. To be more convinced that this solution really corresponds to the brane-antibrane pair, we calculate the tension of the solution using the formula (3.26) with $f^{(0,0)}(\Phi_0)$ replaced by $f^{\left(\frac{3}{2},3\right)}(\Phi_0)$. Marvelously, the resulting value of the ratio has turned out to be

$$r \simeq 0.988.$$

That is to say, the tension of the solution (3.27) agrees with the expected answer to an accuracy of about 1.2%!

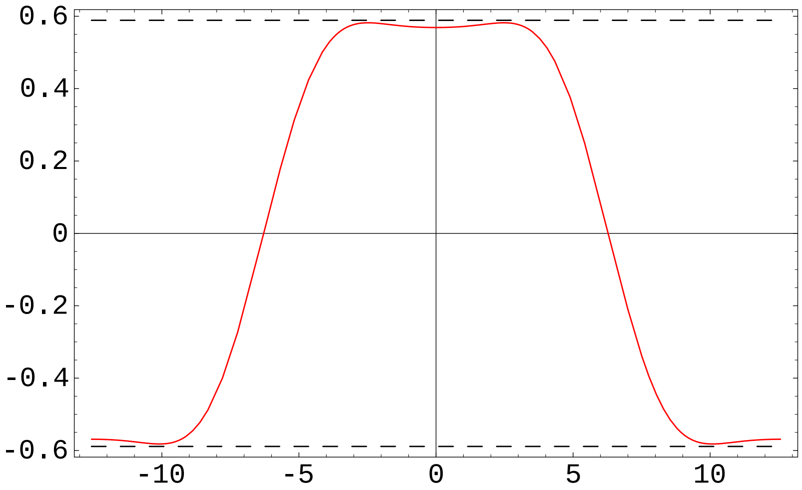

Figure 5: The solid line shows a plot of $t(x)$ found in the level $\left(\frac{25}{16}, \frac{25}{8}\right)$ truncated superstring field theory. The dashed lines indicate the vacuum expectation values $\pm\bar{t} = \pm 0.58882$ of the tachyon field at the minima of the level $\left(\frac{3}{2}, 3\right)$ tachyon potential. $t(x) = 0$ occurs at $x = \pm 2\pi$ for $R = 4$ (in units of $\sqrt{\alpha'}$).

3.2.3 Deformation of the solution

Since we have constructed the multi-brane solution, we want to study the relative motion of constituent branes. (The center-of-mass motion was frozen out when we set $t(x)$ to an even function of x.) Here we insist that the interaction between a kink and an antikink should not be incorporated into the classical solutions of superstring field theory. The reason is as follows: The interaction between two D-branes comes about from the exchange of closed strings. From the point of view of open strings, this corresponds to a 1-loop diagram with no vertex operators inserted. Such loop effects of open strings, however, are not taken into account in superstring field theory at the *classical* level because $\langle\langle \ldots \rangle\rangle$ in the action are evaluated as *disk* correlation functions among the vertex operators inserted. Loop corrections could be calculated if we expanded the string field $\widehat{\Phi}$ around our brane solution $\widehat{\Phi}_\ell$ as $\widehat{\Phi} = \widehat{\Phi}_\ell + \widehat{\Psi}$ and applied the *quantum* superstring field theory to the fluctuation field $\widehat{\Psi}$. Provided that our claim is true, there will be a flat direction in the potential in which the branes can move freely. As can be seen from the fact, however, that one isolated set (3.27) of field values was singled out as a solution to the equations of motion, the level truncation approximation generically lifts a flat direction to an approximate one. In fact, the effective potential for the massless marginal deformation parameter in string field theory was studied in [24, 17] and the authors concluded that the potential was not exactly flat to any given finite order in the level truncation scheme, though it became flatter as the level of approximation was increased. Therefore the kink-antikink configuration, when the distance between two branes is changed arbitrarily, will not solve the equations of motion. Nevertheless, we can 'construct' such configurations by turning on the τ_2 mode by hand. That is, we fix the value of τ_2, say $\tau_2 = 0.1$, and then extremize the action with respect to the remaining 23 variables. Some of the resulting configurations are plotted in figure 6. Though these configurations with $\tau_2 \neq 0$ are not solutions, we can calculate their tensions anyway. The ratio r of the tension of the kink-antikink to twice the tension of the BPS D$(p-1)$-brane, together with the location x_0 of the antikink defined to be a zero of the tachyon profile $t(x)$ with $t'(x_0) < 0$, is shown in table 2 as functions of the deformation parameter τ_2.

τ_2	r (tension)	x_0 (zero of $t(x)$)
0	0.987919	6.283
0.01	0.987768	6.144
0.02	0.987306	6.003
0.03	0.986504	5.859
0.04	0.985314	5.710
0.05	0.983531	5.555
0.06	0.981496	5.392
0.07	0.978685	5.219
0.08	0.975120	5.034
0.09	0.970652	4.831
0.10	0.964936	4.603
0.11	0.958044	4.325
≥ 0.12	no solutions	no solutions

Table 2: We show the value of the ratio r of the tension of the kink-antikink to twice the BPS D$(p-1)$-brane tension, and the location x_0 of the antikink as functions of the deformation parameter τ_2. We found no solutions once τ_2 reached or went beyond 0.12.

Notice that $2x_0$ is in fact equal to the distance between the kink and the antikink due to the symmetry $t(x) = t(-x)$. For the same reason, a configuration for a negative value of τ_2 is equivalent to that for $|\tau_2|$, so we wrote the results only for $\tau_2 \geq 0$ in table 2. Our numerical algorithm ceased to converge when we chose the value equal to or more than 0.12 for τ_2. From this table, one can find that the tension of the kink-antikink system gets *lowered* as the kink and the antikink come closer. In order to find out more detailed relation between the tension r and the distance x_0, we have plotted $\log_{10}(2\pi - x_0)$ versus $\log_{10}(0.987919 - r)$ in figure 7 and fitted these data with a straight line. The result is

$$\log_{10}(0.987919 - r) = -2.09383 + 2.01593 \log_{10}(2\pi - x_0) \tag{3.28}$$

or

$$r = 0.987919 - 0.00805649 \times (2\pi - x_0)^{2.01593} \,. \tag{3.29}$$

As is seen from figure 7, it nicely interpolates the data. We can read from the relation (3.29) that the tension of the system almost quadratically depends on the 'co-distance' $2\pi - x_0$ between the kink and the antikink. Though we do not understand the precise meaning of the exponent '2' appearing in eq. (3.29), it might have some physical meaning since the data points are well approximated by a straight line. One may possibly think that it is a manifestation of the attractive force between a brane and an antibrane. However, we do not think this to be the case. In addition to

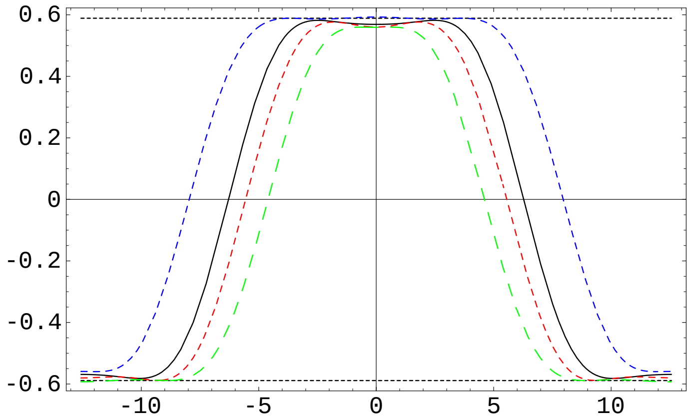

Figure 6: The solid line shows a plot of $t(x)$ in the original kink-antikink solution. The dashed curves represent the translated kink-antikink pairs obtained by setting $\tau_2 = 0.1, 0.05, -0.1$ from below.

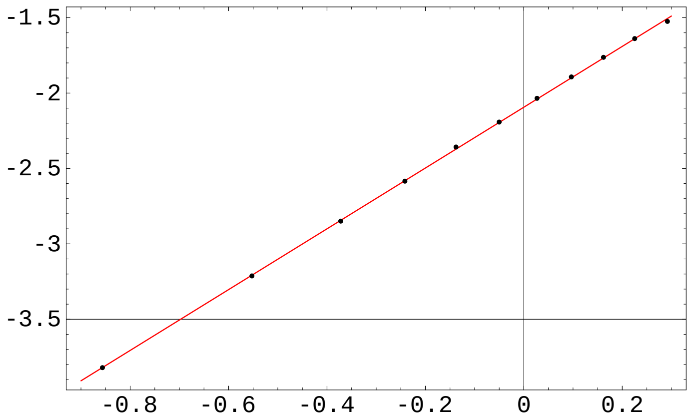

Figure 7: This figure shows the relation between the ratio r and the half-distance x_0. The horizontal axis represents the value of $\log_{10}(2\pi - x_0)$, while the vertical axis $\log_{10}(0.987919 - r)$. The dots are taken from table 2, and the solid line shows the plot of the best fit (3.28).

our claim made at the beginning of this sub-subsection, if the lifting of the potential originated from the attractive force, the x_0-dependence (3.29) of the potential would depend on the dimensionality $(p-1)$ of the branes which interact via the exchange of closed strings. The fact that the formula (3.29) does not contain the dimensionality at all implies that the lifting of the potential has nothing to do with the attractive force carried by the closed strings and is presumably an artifact of our level truncation approximation. In any case, what we have learned from the above analysis are:

- A kink and an antikink are automatically placed at diametrically opposite points of the circle if we solve the full set of equations of motion.
- By setting τ_2 to some fixed values and solving the equations of motion with respect to the other variables, we can deform the above kink-antikink solution

along the quasi-flat direction of the potential representing the relative motion of the branes.

- It seems that the tension of the kink-antikink system is lowered as the kink approaches the antikink.

4. Summary

In this paper we have constructed a kink solution and a kink-antikink solution by applying the level truncation method to the Berkovits' version of open superstring field theory. The tensions of the solutions have agreed with those of the expected BPS D-branes with a remarkable precision (95% and 99% respectively). We have also tried to see the quasi-flat direction of the potential corresponding to the relative motion of the kink and the antikink, with some questions unanswered.

We have focused our attention only on the classical configurations of the string field on a non-BPS D-brane. It would be interesting to study the fluctuation spectra around these solutions, as well as to construct a vortex solution after the tachyon condensation on a coincident brane-antibrane pair.

Acknowledgments

I am grateful to Prof. Tohru Eguchi for his insightful comments and careful reading of the manuscript. I would also like to thank Teruhiko Kawano, Seiji Terashima, Tadashi Takayanagi, Tadaoki Uesugi, Kent Ichikawa and Kazuhiro Sakai for valuable discussions.

A. Transformation of vertex operators

A.1 Conformal transformations

$$g_k^{(n)} \circ (\xi c e^{-\phi} e^{ipX})(0) = \left| \left(\frac{4}{n}\right)^{\alpha' p^2 - \frac{1}{2}} \right| e^{-\pi i \left(\frac{k-1}{n} + \frac{1}{4}\right)} \xi c e^{-\phi} e^{ipX} \left(g_k^{(n)}(0) \right), \quad \text{(A.1)}$$

$$\begin{aligned} g_k^{(n)} \circ (\xi \eta e^{ipX})(0) &= \left| \left(\frac{4}{n}\right)^{\alpha' p^2 + 1} \right| e^{2\pi i \left(\frac{k-1}{n} + \frac{1}{4}\right)} \left(\xi\eta - \frac{g_k^{(n)\prime\prime}(0)}{2(g_k^{(n)\prime}(0))^2} \right) \times \\ &\quad \times e^{ipX} \left(g_k^{(n)}(0) \right) \\ &= \left| \left(\frac{4}{n}\right)^{\alpha' p^2 + 1} \right| e^{2\pi i \left(\frac{k-1}{n} + \frac{1}{4}\right)} \times \\ &\quad \times \left(\xi\eta - \frac{1}{2} e^{-2\pi i \frac{k-1}{n}} \right) e^{ipX} \left(g_k^{(n)}(0) \right), \end{aligned} \quad \text{(A.2)}$$

$$g_k^{(n)} \circ (\xi\partial\xi c\partial^2 ce^{-2\phi}e^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2+1}\right| e^{2\pi i\left(\frac{k-1}{n}+\frac{1}{4}\right)} \times \tag{A.3}$$

$$\times\xi\partial\xi\left(c\partial^2 c - e^{-2\pi i\frac{k-1}{n}}c\partial c\right)e^{-2\phi}e^{ipX}\left(g_k^{(n)}(0)\right),$$

$$g_k^{(n)} \circ (\xi c(\partial e^{-\phi})\psi^X e^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2+1}\right| e^{2\pi i\left(\frac{k-1}{n}+\frac{1}{4}\right)} \times$$

$$\times\xi c\left(\partial e^{-\phi} + \frac{1}{2}e^{-2\pi i\frac{k-1}{n}}e^{-\phi}\right)\psi^X e^{ipX}\left(g_k^{(n)}\right), \tag{A.4}$$

$$g_k^{(n)} \circ (\xi ce^{-\phi}G^{\mathcal{M}}e^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2+1}\right| e^{2\pi i\left(\frac{k-1}{n}+\frac{1}{4}\right)}\xi ce^{-\phi}G^{\mathcal{M}}e^{ipX}\left(g_k^{(n)}(0)\right), \tag{A.5}$$

$$g_k^{(n)} \circ (\xi ce^{-\phi}\partial\psi^X e^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2+1}\right| e^{2\pi i\left(\frac{k-1}{n}+\frac{1}{4}\right)} \times \tag{A.6}$$

$$\times\xi ce^{-\phi}\left(\partial\psi^X + \frac{1}{2}e^{-2\pi i\frac{k-1}{n}}\psi^X\right)e^{ipX}\left(g_k^{(n)}(0)\right),$$

$$g_k^{(n)} \circ (\xi ce^{-\phi}\psi^X\partial Xe^{ipX})(0) = \left|\left(\frac{4}{n}\right)^{\alpha' p^2+1}\right| e^{2\pi i\left(\frac{k-1}{n}+\frac{1}{4}\right)} \times \tag{A.7}$$

$$\times\xi ce^{-\phi}\psi^X\left(\partial X - i\alpha' pe^{-2\pi i\frac{k-1}{n}}\right)e^{ipX}\left(g_k^{(n)}(0)\right).$$

In the above lines, we used (2.20), (2.21) so that

$$\frac{g_k^{(n)\prime\prime}(0)}{(g_k^{(n)\prime}(0))^2} = e^{-2\pi i\frac{k-1}{n}}.$$

A.2 BRST transformations

$$Q_0(\xi ce^{-\phi}e^{ipX}) = \left(\alpha' p^2 - \frac{1}{2}\right)\xi c\partial ce^{-\phi}e^{ipX}, \tag{A.8}$$

$$Q_1(\xi ce^{-\phi}e^{ipX}) = -\sqrt{2\alpha'}p\ c\psi^X e^{ipX}, \tag{A.9}$$

$$Q_2(\xi ce^{-\phi}e^{ipX}) = -\eta e^{\phi}e^{ipX}, \tag{A.10}$$

$$Q_0(\xi\eta e^{ipX}) = \alpha' p^2\partial c\xi\eta e^{ipX} + \partial(c\xi\eta e^{ipX}) - \frac{1}{2}\partial^2 c\ e^{ipX}, \tag{A.11}$$

$$Q_1(\xi\eta e^{ipX}) = \eta e^{\phi}G^{\mathrm{m}}e^{ipX} + \sqrt{2\alpha'}p\ \eta\partial(e^{\phi}\psi^X)e^{ipX}, \tag{A.12}$$

$$Q_2(\xi\eta e^{ipX}) = -2\eta\partial\eta e^{2\phi}be^{ipX}, \tag{A.13}$$

$$Q_1(\partial\xi c\partial^2 ce^{-2\phi}e^{ipX}) = \sqrt{2\alpha'}p\ c\partial^2 ce^{-\phi}\psi^X e^{ipX}, \tag{A.14}$$

$$Q_2(\partial\xi c\partial^2 ce^{-2\phi}e^{ipX}) = (6\partial\eta + 8\partial\phi\ \eta)ce^{ipX}, \tag{A.15}$$

$$Q_0(c\partial e^{-\phi}\psi^X e^{ipX}) = (\alpha' p^2+1)\partial cc\partial e^{-\phi}\psi^X e^{ipX} + \frac{1}{2}\partial^2 cce^{-\phi}\psi^X e^{ipX}, \tag{A.16}$$

$$Q_2\left(\xi c(\partial e^{-\phi})\psi^X e^{ipX}\right) = (5\eta\partial e^{\phi} + 4\partial\eta e^{\phi} + 2\eta bce^{\phi})\psi^X e^{ipX}, \tag{A.17}$$

$$Q_2(\xi ce^{-\phi}G^{\mathcal{M}}e^{ipX}) = e^{\phi}\eta G^{\mathcal{M}}e^{ipX}, \tag{A.18}$$

$$Q_2(\xi c e^{-\phi}(\partial\psi^X)e^{ipX}) = e^{\phi}\eta(\partial\psi^X)e^{ipX}\,, \tag{A.19}$$
$$Q_2(\xi c e^{-\phi}\psi^X\partial X e^{ipX}) = e^{\phi}\eta\psi^X\partial X e^{ipX}\,. \tag{A.20}$$

B. Sample calculations

In this section we exhibit some computations of the action. Firstly, at the level of approximation used in this paper, it is sufficient to keep only those terms with up to six string fields in the action (2.1) due to the conservation of ϕ-charge and the level truncation [14]. We can further simplify the action by appealing to the twist symmetry and cyclicity properties (2.25), the result being

$$\begin{aligned} S = \frac{1}{2g_o^2}\Big\langle\!\!\Big\langle &\frac{1}{2}(\widehat{Q}_B\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{3}(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{12}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi}) - \widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\right) + \\ &+ \frac{1}{60}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi}) - 3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\right) + \\ &+ \frac{1}{360}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^4(\widehat{\eta}_0\widehat{\Phi}) - 4\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi} + 3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}^2\right)\Big\rangle\!\!\Big\rangle\,. \end{aligned} \tag{B.1}$$

At the purely tachyonic level, $\widehat{\Phi} = T\otimes\sigma_1$, the nonvanishing terms are

$$\begin{aligned} 2g_o^2 S_T &= \frac{1}{2}\Big\langle\!\!\Big\langle\left(\widehat{Q}_B\widehat{\Phi}\right)\left(\widehat{\eta}_0\widehat{\Phi}\right)\Big\rangle\!\!\Big\rangle + \frac{1}{12}\Big\langle\!\!\Big\langle\left(\widehat{Q}_B\widehat{\Phi}\right)\widehat{\Phi}^2\left(\widehat{\eta}_0\widehat{\Phi}\right) - \left(\widehat{Q}_B\widehat{\Phi}\right)\widehat{\Phi}\left(\widehat{\eta}_0\widehat{\Phi}\right)\widehat{\Phi}\Big\rangle\!\!\Big\rangle \\ &= -\left\langle g_1^{(2)}\circ(Q_0T)\; g_2^{(2)}\circ(\eta_0 T)\right\rangle - \\ &\quad -\frac{1}{6}\left\langle g_1^{(4)}\circ(Q_2T)\; g_2^{(4)}\circ T\; g_3^{(4)}\circ T\; g_4^{(4)}\circ(\eta_0T)\right\rangle - \\ &\quad -\frac{1}{6}\left\langle g_1^{(4)}\circ(Q_2T)\; g_2^{(4)}\circ T\; g_3^{(4)}\circ(\eta_0T)\; g_4^{(4)}\circ T\right\rangle\,, \end{aligned}$$

where we have taken the trace and used the decomposition (2.2) of the BRST charge. Let us show how to calculate them. Since the BRST charge Q_B and η_0 commute with the Virasoro generators $L_n^{\rm tot}$, we can conformally transform T in advance, namely

$$g_k^{(n)}\circ(Q_BT) = Q_B\left(g_k^{(n)}\circ T\right),\quad g_k^{(n)}\circ(\eta_0T) = \eta_0\left(g_k^{(n)}\circ T\right).$$

Using the definition (2.35) of T and the partial integration (2.26), the 2-point correlator becomes

$$\begin{aligned} -\left\langle g_1^{(2)}\circ(Q_0T)\; g_2^{(2)}\circ(\eta_0T)\right\rangle = -\sum_{m,n} t_m t_n &\left|\left(\frac{4}{2}\right)^{\frac{\alpha'}{R^2}(m^2+n^2)-1}\right| e^{-\pi i\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)}\times \\ &\times\left\langle \xi c e^{-\phi}e^{i\frac{m}{R}X}(z_1)Q_0\left(ce^{-\phi}e^{i\frac{n}{R}X}\right)(z_2)\right\rangle\,, \end{aligned} \tag{B.2}$$

where we have used the conformal transformation (A.1) for $n=2$, and defined

$$z_i = g_i^{(2)}(0) = \begin{cases} 1 & \text{for } i=1 \\ -1 & \text{for } i=2 \end{cases}.$$

The correlator in (B.2) is calculated as

$$\left(\frac{\alpha'}{R^2}n^2-\frac{1}{2}\right)\left\langle\xi(z_1)c(z_1)\partial cc(z_2)e^{-\phi(z_1)}e^{-\phi(z_2)}e^{i\frac{m}{R}X(z_1)}e^{i\frac{n}{R}X(z_2)}\right\rangle=$$
$$=\left(\frac{\alpha'}{R^2}n^2-\frac{1}{2}\right)\frac{(z_1-z_2)^2}{2}\frac{1}{z_1-z_2}\langle\xi(z_1)\partial^2c\partial cc(z_2)e^{-2\phi(z_2)}\rangle_{\rm ghost}|z_1-z_2|^{\frac{2\alpha'}{R^2}mn}\times$$
$$\times\, 2\pi RV_p\delta_{m+n,0}=$$
$$=-2\pi RV_p\delta_{m+n,0}\cdot 2^{-\frac{2\alpha'}{R^2}n^2+1}\left(\frac{\alpha'}{R^2}n^2-\frac{1}{2}\right),$$

where we have used the action (A.8) of Q_0 on T, the OPEs (2.10), (2.5), the normalization (2.24) and

$$\langle e^{iqX}\rangle_{\rm matter}=(2\pi)^{p+1}\delta^{p+1}(q)$$

on a Dp-brane. Multiplying it by the prefactors of (B.2), we finally obtain the expression for the quadratic term in t as

$$2g_o^2S_T^{(2)}=2\pi RV_p\sum_n\left(-\frac{\alpha'}{R^2}n^2+\frac{1}{2}\right)t_{-n}t_n\,. \qquad \text{(B.3)}$$

Now we turn to the quartic terms. We need to calculate

$$\begin{aligned}\mathcal{A}(a,b)&=\langle g_1^{(4)}\circ(Q_2T)\ g_2^{(4)}\circ T\ g_a^{(4)}\circ T\ g_b^{(4)}\circ(\eta_0T)\rangle\\
&=\sum t_{n_1}t_{n_2}t_{n_a}t_{n_b}\left|\left(\frac{4}{4}\right)^{\frac{\alpha'}{R^2}(n_1^2+n_2^2+n_a^2+n_b^2)-2}\right|e^{-\pi i\frac{1+2+3+4}{4}}\times\\
&\quad\times\left\langle Q_2\left(\xi ce^{-\phi}e^{i\frac{n_1}{R}X}\right)(z_1)\xi ce^{-\phi}e^{i\frac{n_2}{R}X}(z_2)\xi ce^{-\phi}e^{i\frac{n_a}{R}X}(z_a)ce^{-\phi}e^{i\frac{n_b}{R}X}(z_b)\right\rangle\\
&=\sum t_{n_1}t_{n_2}t_{n_a}t_{n_b}e^{-\frac{\pi i}{2}}\left\langle(-\eta e^{\phi})(z_1)\xi ce^{-\phi}(z_2)\xi ce^{-\phi}(z_a)ce^{-\phi}(z_b)\right\rangle_{\rm g}\times\\
&\quad\times\left\langle e^{i\frac{n_1}{R}X_1}e^{i\frac{n_2}{R}X_2}e^{i\frac{n_a}{R}X_a}e^{i\frac{n_b}{R}X_b}\right\rangle_{\rm m}\\
&=-i\sum t_{n_1}t_{n_2}t_{n_a}t_{n_b}\langle\eta_1\xi_2\xi_a(-1)c_2c_ac_be^{\phi_1}e^{-\phi_2}e^{-\phi_a}e^{-\phi_b}\rangle_{\rm g}\times\\
&\quad\times 2\pi RV_p\delta_{n_1+n_2+n_a+n_b,0}|z_{12}|^{\frac{2\alpha'}{R^2}n_1n_2}|z_{1a}|^{\frac{2\alpha'}{R^2}n_1n_a}|z_{1b}|^{\frac{2\alpha'}{R^2}n_1n_b}|z_{2a}|^{\frac{2\alpha'}{R^2}n_2n_a}\times\\
&\quad\times|z_{2b}|^{\frac{2\alpha'}{R^2}n_2n_b}|z_{ab}|^{\frac{2\alpha'}{R^2}n_an_b}\,,\end{aligned}$$

where we have defined

$$z_j=g_j^{(4)}(0)=\begin{cases}1 & \text{for } j=1\\ i & \text{for } j=2\\ -1 & \text{for } j=3\\ -i & \text{for } j=4\end{cases},$$

and $z_{jk}=z_j-z_k$. The subscript of the field denotes its position: e.g. $\xi_a=\xi(z_a)$. The ghost correlator can be evaluated as

$$-\left\langle\left(\frac{\xi_a}{z_{12}}-\frac{\xi_2}{z_{1a}}\right)\cdot z_{2a}z_{2b}z_{ab}\frac{1}{2}\partial^2c\partial cc\cdot\frac{z_{12}z_{1a}z_{1b}}{z_{2a}z_{2b}z_{ab}}e^{-2\phi}\right\rangle_{\rm g}=z_{2a}z_{1b} \qquad \text{(B.4)}$$

using the relevant OPEs. Then we find

$$\mathcal{A}(3,4)=2\times 2\pi RV_p\sum\delta_{n_1+n_2+n_3+n_4}t_{n_1}t_{n_2}t_{n_3}t_{n_4}\cdot 2^{\frac{\alpha'}{R^2}(n_1n_2+n_2n_3+n_3n_4+n_4n_1+2n_1n_3+2n_2n_4)},$$
$$\mathcal{A}(4,3)=4\times 2\pi RV_p\sum\delta_{n_1+n_2+n_3+n_4}t_{n_1}t_{n_2}t_{n_3}t_{n_4}\cdot 2^{\frac{\alpha'}{R^2}(n_1n_2+n_2n_3+n_3n_4+n_4n_1+2n_1n_3+2n_2n_4)}.$$

The quartic terms in the action can be written as

$$\begin{aligned}2g_o^2S_T^{(4)} &= -\frac{1}{6}\mathcal{A}(3,4)-\frac{1}{6}\mathcal{A}(4,3)\\ &= -2\pi RV_p\sum\delta_{n_1+n_2+n_3+n_4}t_{n_1}t_{n_2}t_{n_3}t_{n_4}\cdot 2^{\frac{\alpha'}{R^2}((n_1+n_3)^2-n_1^2-n_2^2-n_3^2-n_4^2)},\end{aligned} \tag{B.5}$$

where the exponent was put to the above form using the momentum conservation. Combining eq. (B.3) and (B.5), we finally get the purely tachyonic string field theory action (3.8).

Of course, we must also calculate a large number of correlators which contain non-tachyonic modes, even though only those terms with -2 units of total ϕ-charge will survive. As an example, we will illustrate the calculations with the following quartic coupling

$$\mathcal{B}(a,b,c,d)=\left\langle g_a^{(4)}\circ(Q_1T)\; g_b^{(4)}\circ T\; g_c^{(4)}\circ E\; g_d^{(4)}\circ(\eta_0 J)\right\rangle .$$

Using the conformal transformation laws (A.1), (A.2), (A.7) and the action (A.9) of Q_1 on T, it can be calculated as

$$\begin{aligned}\mathcal{B}(a,b,c,d) &= \frac{i}{\sqrt{\alpha'}}\sum t_{n_a}t_{n_b}e_{n_c}j_{n_d}\left|\left(\frac{4}{4}\right)^{\frac{\alpha'}{R^2}(n_a^2+n_b^2+n_c^2+n_d^2)+1}\right|e^{\frac{2\pi i}{4}\left(-\frac{a}{2}-\frac{b}{2}+c+d\right)}\times\\ &\quad\times\left\langle Q_1\left(\xi ce^{-\phi}e^{i\frac{n_a}{R}X}\right)(z_a)\xi ce^{-\phi}e^{i\frac{n_b}{R}X}(z_b)\left(\xi\eta(z_c)-\frac{1}{2}e^{\frac{\pi i}{2}(1-c)}\right)\times\right.\\ &\quad\left.\times e^{i\frac{n_c}{R}X(z_c)}ce^{-\phi}\psi^X\left(\partial X-i\frac{\alpha'}{R}n_de^{\frac{\pi i}{2}(1-d)}\right)e^{i\frac{n_d}{R}X}(z_d)\right\rangle\\ &= \frac{i}{\sqrt{\alpha'}}\sum t_{n_a}t_{n_b}e_{n_c}j_{n_d}e^{\frac{\pi i}{2}\left(-5+\frac{3}{2}(c+d)\right)}\left(-\frac{\sqrt{2\alpha'}}{R}n_a\right)\times\\ &\quad\times\left\langle c\psi^X(z_a)\xi ce^{-\phi}(z_b)\left(\xi\eta(z_c)-\frac{1}{2}e^{\frac{\pi i}{2}(1-c)}\right)ce^{-\phi}\psi^X(z_d)\right\rangle_{\text{ghost},\psi}\times\\ &\quad\times\left\langle\left(\partial X(z_d)-i\frac{\alpha'}{R}n_de^{\frac{\pi i}{2}(1-d)}\right)e^{i\frac{n_a}{R}X_a}e^{i\frac{n_b}{R}X_b}e^{i\frac{n_c}{R}X_c}e^{i\frac{n_d}{R}X_d}\right\rangle_X ,\end{aligned}$$

where we have arranged the phase factor in the above way using the fact $a+b+c+d=10$. ($\{a,b,c,d\}$ is a permutation of $\{1,2,3,4\}$.) Taking care not to forget the minus signs when fermionic fields pass each other, the ghost- and ψ-part becomes

$$-\left\langle\xi_b\left(:\xi_c\eta_c:-\frac{1}{2}e^{\frac{\pi i}{2}(1-c)}\right)c_ac_bc_de^{-\phi_b}e^{-\phi_d}\psi_a^X\psi_d^X\right\rangle$$

$$
\begin{aligned}
&= -\left\langle \left(-\frac{\xi_c}{z_{bc}} - \frac{1}{2} e^{\frac{\pi i}{2}(1-c)} \xi_b \right) \cdot z_{ab} z_{ad} z_{bd} \frac{1}{2} \partial^2 c \partial c c \cdot \frac{1}{z_{bd}} e^{-2\phi} \cdot \frac{1}{z_{ad}} \right\rangle \\
&= -\left(\frac{1}{z_{bc}} + \frac{1}{2} e^{\frac{\pi i}{2}(1-c)} \right) z_{ab}.
\end{aligned}
$$

For the X-part, the OPE (2.6) gives

$$
\begin{aligned}
&\left\{ \frac{i}{R}(-2\alpha') \left(\frac{n_a}{z_{da}} + \frac{n_b}{z_{db}} + \frac{n_c}{z_{dc}} \right) - i \frac{\alpha'}{R} n_d e^{\frac{\pi i}{2}(1-d)} \right\} 2\pi R V_p \delta_{n_a+n_b+n_c+n_d} \times \\
&\times |z_{ab}|^{\frac{2\alpha'}{R^2} n_a n_b} p |z_{ac}|^{\frac{2\alpha'}{R^2} n_a n_c} |z_{ad}|^{\frac{2\alpha'}{R^2} n_a n_d} |z_{bc}|^{\frac{2\alpha'}{R^2} n_b n_c} |z_{bd}|^{\frac{2\alpha'}{R^2} n_b n_d} |z_{cd}|^{\frac{2\alpha'}{R^2} n_c n_d} = \\
&\quad = -i \frac{\alpha'}{R} \left(\frac{2n_a}{z_{da}} + \frac{2n_b}{z_{db}} + \frac{2n_c}{z_{dc}} + n_d e^{\frac{\pi i}{2}(1-d)} \right) 2\pi R V_p \delta_{\Sigma n} \; 2^{\frac{\alpha'}{R^2}\left\{ (n_1+n_3)^2 - \Sigma n^2 \right\}}.
\end{aligned}
$$

Collecting these factors, we finally obtain

$$
\begin{aligned}
\mathcal{B}(a,b,c,d) = 2\pi R V_p \frac{\sqrt{2}\alpha'}{R^2} z_{ab} &\left(\frac{1}{z_{bc}} + \frac{1}{2} e^{\frac{\pi i}{2}(1-c)} \right) e^{\frac{\pi i}{2}\left(-5+\frac{3}{2}(c+d)\right)} \times \\
&\times \sum_{\{n\}} \delta_{\Sigma n} \; 2^{\frac{\alpha'}{R^2}\left\{ (n_1+n_3)^2 - \Sigma n^2 \right\}} n_a t_{n_a} t_{n_b} e_{n_c} j_{n_d} \times \\
&\times \left(\frac{2n_a}{z_{da}} + \frac{2n_b}{z_{db}} + \frac{2n_c}{z_{dc}} + n_d e^{\frac{\pi i}{2}(1-d)} \right).
\end{aligned}
$$

Repeating the computations this way, one could write down the component form of the action. Concerning our result,

1. our action reproduces the level $(3/2, 3)$ tachyon potential found in [14] when we restrict all fields in the action to zero momentum ($n = 0$) sector, and

2. our action is manifestly real if the component fields defined in (2.35) are real.

These features suggest that our action is correct. In particular, the fact 1. strongly supports the correctness of our calculations in the ghost sector.

C. On the use of conservation laws

In ref. [25], a convenient computational scheme, using the conservation laws, for string field theory has been developed. Although the authors of [25] made a brief comment on the applicability of the conservation laws to superstring field theory, we have judged that it is not very advantageous to use the conservation laws for calculating the string vertices in Berkovits' superstring field theory for the following reasons.

The first point, though not so problematic, is that every string vertex contains the insertion of the BRST charge Q_B, as opposed to the cubic vertex in bosonic cubic string field theory [2]. Since it takes the complicated form

$$Q_B^{\text{holom.}} = \sum_m c_{-m} L_m^{\mathrm{m}} + \sum_r \gamma_{-r} G_r^{\mathrm{m}} - \sum_{m,n} \frac{n-m}{2} {}^{\circ}_{\circ} b_{-m-n} c_m c_n {}^{\circ}_{\circ} + \\ + \sum_{m,r} \left[\frac{2r-m}{2} {}^{\circ}_{\circ} \beta_{-m-r} c_m \gamma_r {}^{\circ}_{\circ} - {}^{\circ}_{\circ} b_{-m} \gamma_{m-r} \gamma_r {}^{\circ}_{\circ} \right] + a^{\mathrm{g}} c_0$$

in the operator representation, we want to avoid making explicit use of it. The second point is the problem of dealing with the states in the "large" Hilbert space. If we were working in the "small" Hilbert space, it would be most convenient to write down the relevant conservation laws for the β, γ oscillators. However, once we carry out the bosonization (2.4) of the β, γ ghosts in order to incorporate the zero mode of ξ, we will be confronted with the treatment of the factor $e^{\ell\phi}$. Since we are not given an oscillator form of it, we have to handle it in a similar way as the momentum factor e^{ipX}. We are not familiar with such a formulation.

The third problem consists in the way of finding the conservation laws. In the case of bosonic cubic string field theory, the Virasoro conservation laws of the general form

$$\langle V_3 | L_{-k}^{(2)} = \langle V_3 | \left(A^k \cdot c + \sum_{n \geq 0} a_n^k L_n^{(1)} + \sum_{n \geq 0} b_n^k L_n^{(2)} + \sum_{n \geq 0} c_n^k L_n^{(3)} \right) \tag{C.1}$$

have been derived in [25] by introducing a holomorphic vector field $v(z)$. To obtain the Virasoro conservation law for the $k = 2$ case, we had to choose the vector field to be

$$v(z) = \frac{(z - z_1)(z - z_3)}{z - z_2}, \tag{C.2}$$

where z_i denotes the insertion point of the i-th vertex operator on the boundary of the global disk. That is, $v(z)$ has zeroes at punctures 1 and 3, and has a simple pole at puncture 2. If we wish to extend the conservation law (C.1) for $k = 2$ to, say, the 5-point vertex $\langle V_5|$, it is expected that the holomorphic vector field must be of the form

$$v(z) = \frac{(z - z_1)(z - z_3)(z - z_4)(z - z_5)}{z - z_2}.$$

However, it does not respect the holomorphicity at infinity, which is expressed as

$$\lim_{z \to \infty} z^{-2} v(z) < +\infty .$$

Hence it seems that it is impossible to write down Virasoro conservation laws with a small value of k for the string vertices of sufficiently high order.

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